Scottish Electrical Charitable Training Trust

Open Learning

An Introduction to Mathematics

UNIT 1

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This unit has been prepared by SECTT to be used by Electrotechnical candidates undertaking the Scottish Joint Industry Board (SJIB) Training Programmes.

There are 3 units in the set of mathematical support packages, which are designed as student centred, or tutor guided learning support materials. They are designed for candidates who may require some additional support and development of their mathematical skills and ability.

The use of this unit is restricted to such candidates.

SECTT will only provide copies of units free of charge to those candidates who would benefit from this course of study, and assist in their achievement of the required standard in practical and theoretical course work.

Welcome to your Open Learning Unit
An Introduction to Mathematics

This unit will build on your previous experience and help you to attain the mathematical skills necessary in a variety of technology areas. All topics are dealt with from the start and no previous experience is assumed.

However, it is largely revision of basic skills so no calculator to be used. Consequently you need to know your tables to complete this unit as is the case in the S.E.C.T.T. Pre-employment Assessment.

The later two units will require the use of a scientific calculator as in your College course and the world of work.

The topics covered in this unit are :

- Addition, subtraction, multiplication and division of whole numbers
- Addition, subtraction, multiplication and division of decimal numbers
- Addition, subtraction, multiplication and division of fractions
- Squares, square roots and reciprocals
- Percentages
- Algebra including use of formulae

Guide to Study

While the unit has been written to allow you to progress on your own as a self-study pack, you will benefit from certain basic rules:

1. You should plan your study time by working out when and for how long you can study each week. When you have made a plan, stick to it.

2. Do not plan to take on too much studying at once by limiting yourself to one or two hours at a time. Try to finish with a topic and self assessed questions (SAQs).

3. At the start of the following study session briefly go over your previous piece of work.

4. Do not get "stuck" at a particular topic, or this will dishearten and discourage you. If you are stuck seek assistance or help from your tutor.

Answers for all questions are provided at the back of the unit. Do not go to the answers until you have attempted the question. This unit is for you, and has been designed to help and assist you.
IF YOU CHEAT, YOU ARE ONLY CHEATING YOURSELF!

THE NUMBER SYSTEM

The most common number system uses 10 as a base. The numbers used are 1, 2, 3, 4, 5, 6, 7, 8 and 9. With a plus sign in front they are called positive numbers; with a minus sign they are called negative numbers.
A good way of viewing these is as a number line :

-9 -8 -7 -6 -5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5 +6 +7 +8 + 9

Note

The number 0 (zero) is neither positive nor negative.
The positive numbers do not usually have the + sign written in front of them.

Thus :

-9 -8 -7 -6 -5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5 +6 +7 +8 + 9

← Increasing negatively   increasing positively →

As you will know, beyond 9 we progress as follows:
9, 10, 11, 12, 13, 14, 15, 16, 17, 18, - - - 22 - - etc
and
-9, -10, -11, -12, -13, -14, -15, -16, -17, -18, - - - -22 - - etc

ADDITION AND SUBTRACTION

You will need to know basic additions such as 3 + 8 = 11 or be able to work them out.
Note that the + sign is used here in its normal sense of ADD.
In some cases we are asked to ADD 3 to -5 which we could write as 3 + (-5) but usually we would write this as 3 - 5. Here the - sign is being used in its normal sense of SUBTRACT. Thus 3 - 5 = -2 (we must write -2 as the answer is a negative number which must have the minus or negative sign (-) before it).
These two examples have introduced the rules relating to + and - signs when applied to addition and subtraction :

+ + results in a +
+ - results in a -
- + results in a -
- - results in a +

Examples

4 + 3 = 7
4 - 3 = 1
-4 + 3 = -1
-4 + (-3) = -4 - 3 = -7 using the rule of signs
-4 - (-3) = -4 + 3 = -1 using the rule of signs

So far we have only considered single digit numbers like 5, 8, 4 etc.
When multiple digit numbers are involved you may recall using the following layout :

17 + 32 = 49 written as

$$\begin{array}{r} 17 \\ \underline{+32} \\ 49 \end{array}$$

using 7 + 2 = 9 in the units column
and 3 + 1 = 4 in the tens column.

Recall also that it may be necessary to “carry” when 9 is exceeded in addition, or “borrow” when subtracting a larger number from a smaller number :

$$\begin{array}{r} 158 \\ \underline{+249} \\ 407 \end{array}$$

using 9 + 8 = 17, writing 7 in the units column, carrying 1 to the 4 in the tens column making 5 + 5 = 10, writing 0 in the tens column, carrying 1 to the 2 in the hundreds column making 3 then 3 + 1 = 4 in the that column.

Another approach :
158 = 100 + 50 + 8
249 = 200 + 40 + 9 then
100 + 200 = 300,
50 + 40 = 90
8 + 9 = 17 and the answer is 390 + 17 = 407

(390 + 17 = 390 + 10 + 7 = 400 + 7 = 407)

Now recall subtraction :

$$\begin{array}{r} 527 \\ \underline{-\ 318} \\ 209 \end{array}$$

(using 17 - 8 = 9 in the units column, borrowing 1 from the 2 in the first row, then 1 - 1 = 0 in the tens column and and 5 - 3 = 2 in the hundreds column)

Now do Self Assessed Questions Exercise 1 (SAQ1)

SAQ1

Calculate :

1.	2 + 5	2.	12 + 15	3.	6 - 5
4.	6 - 15	5.	8 + (-3)	6.	17 - (-10)
7.	184 + 215	8.	547 + 386	9.	957 - 743
10.	453 - 397	11	8546 - 3891	12.	265 + 123 + 56

Answers on page 35

MULTIPLICATION

The statement 3 x 2 means take the number 3 and multiply it by the number 2.

Alternatively we can write this as 3 x 2 = 3 + 3. Note that 3 x 2 means add the number three two times.

Again 4 x 6 = 4 + 4 + 4 + 4 + 4 + 4 = 24

Obviously if we are asked to perform 8 x 7 the above method is too lengthy so we rely on our knowledge of the multiplication tables.
Hence we proceed by stating 8 x 7 = 56

(For this unit and any test where no calculator is allowed, you should practise writing out multiplication tables if necessary. Even when a calculator is available it is useful to perform mental calculations as a check on answers, also in some multiple choice answer questions as in the test at the end of the unit).

The rules applying to the positive (+) and negative signs for addition and subtraction are the same for multiplication (and division) :

+ x + = +
+ x - = -
- x + = -
- x - = +

i.e. Multiplication of like signs results in a positive (+) and of unlike signs results in a negative.

For instance :

8 x -6 = -48

-8 x -9 = +72 usually just written as 72

-4 x 6 = -24

and of course 7 x 8 = 56

SAQ2

Now carry out the following multiplications :

1.	9 x 5	2.	3 x 8	3.	6 x 9	4.	2 x 8
5.	7 x 6	6.	-5 x -5	7.	6 x 4	8.	-6 x 8
9.	8 x -7	10.	-3 x -2	11	-8 x -8	12.	9 x 9

Answers on page 35

So far in our multiplication we have only considered single digit numbers. Often we are asked to multiply multiple digit numbers such as 26 by 3 for example.

To perform this multiplication, you may use the following layout :

			26	
			x 3	
	3 x 6	=	18	
	3 x 20	=	60	
	Add to obtain		78	as the result.
Similarly	84 x 7	=	84	
			x7	
	7 x 4	=	28	
	7 x 80	=	560	
	Result	=	588	
And	1258 x 5	=	1258	
			x5	
	5 x 8	=	40	
	5 x 50	=	250	
	5 x 200	=	1000	
	5 x 1000	=	5000	
	Result	=	6290	

You may prefer the more condensed form of long multiplication which required “carrying”.
These examples would be written :

26	84	1258
x 3	x 7	x 5
78	588	6290

A multiplication like 641 x 127 would be very long using the first layout so the second is preferred - could you do it?
Obviously a calculator is the tool to use and as this unit is primarily to revise and understand basic skills, calculations like this will not be required in it.

SAQ3

Carry out the following multiplications :

1. 9 x 81
2. 6 x 72
3. 3 x 456
4. 2 x 987
5. 5 x 8792
6. 7 x 1547

Answers on page 35

DIVISION

We now consider the problem of one number (for example 6) being divided by another (for example 2).

This is written 6/2 or 6 ÷ 2

The statement is asking how many times will the number 2 go into the number 6. Clearly, without a calculator at this stage you simply require to go through the 2 times tables to find that 6 ÷ 2 = 3 because 2 x 3 = 6.

Similarly 15 ÷ 5 = 3 (5 times tables)
27 ÷ 9 = 3 (9 times tables)
16 ÷ 8 = 2 (8 times tables)

As indicated earlier, the rules relating to + and - in division are the same as for multiplication. Two similar signs result in a + and two dissimilar signs result in a -.

Example

$6 \div (-2)$ or $\frac{6}{-2} = -3$

$-75 \div 15$ or $\frac{-75}{15} = -5$

$-108 \div -9$ or $\frac{-108}{-9} = 12$

In this package you will only be asked to divide by single digit numbers.
When dividing into a multiple digit number for example $5838 \div 7$ you may recall the layout :

	834	-	Result
Divider -	$7\sqrt{5838}$	-	Dividend
(Subtract)	<u>56</u>	-	(56 from 7 x 8)
	23		
(Subtract)	<u>21</u>	-	(21 from 7 x 3)
	28		
(Subtract)	<u>28</u>	-	(28 from 7 x 4)
	00	-	remainder

SAQ4

Carry out the following multiplications and divisions :

1.	9 x 3	2.	21 x 7	3.	108 x 9	4.	4 ÷ -2
5.	-75 x -5	6.	18 ÷ -9	7.	123 ÷ 3	8.	3296 ÷ 8
9.	4921 ÷ 7	10.	3906 ÷ 9	11.	3906 ÷ 6	12	34245 ÷ 5

Answers on page 35

DECIMAL NUMBERS

Before we can proceed further we must introduce the decimal system
Note that the number 15 can be written as

1 5 . 0 0 0

Tens — Units — Tenths — Hundredths — Thousandths

We will shortly be using numbers such as

1 2 7 . 3 8 9

1 Hundred — 2 Tens — 7 Units — 3 Tenths — 8 Hundredths — 9 Thousandths

which will read as 127 point 389.

Similar to whole numbers, we can add, subtract, multiply and divide decimal numbers both positive and negative. You will recall that the decimal point must be "lined up" in addition and subtraction.

Examples

1. 27.89 + 14.006 would be written as

```
  27.89
 +14.006
 -------
  41.896
```

2. 120.465 - 21.08 would be written as

```
 120.465
  21.08
 -------
  99.385
```

3. 17.58 x 9 would be written as

```
  17.58
      9
 ------
 158 .22
```

Note that the result 158.22 has the decimal point placed as in 17.58, the number being multiplied. We will only be multiplying by single digit whole numbers in this unit.

4. 15 ÷ 6 would be written as

```
     2.5     - result
 6√15.000
   12
     30
     30
    nil      - remainder
```

The process is
(1) 6 into 15 goes 2 (twice sounds better !) and 3 over
(2) Now bring down the first decimal place 0 and put a decimal point in the result.
(3) 6 into 30 goes 5 times with nil remainder
(4) The result is 2.5 (two point five)

Another example 29.65 ÷ 5

The process is

$$\begin{array}{r} 5.93 \\ 5\overline{)29.650} \\ \underline{25} \\ 46 \\ \underline{45} \\ 15 \\ \underline{15} \end{array}$$

nil remainder

SAQ5

Calculate ;

1.	7.82 + 9.37	2.	821.10 + 729.78	3.	6.37 - 2.84
4.	6.532 - 4.05	5.	452.36 + 198.29 + 911.32 + 124.67		
6.	657.62 - 385.37	7.	5.2 x 7	8.	49.65 x 8
9.	218.37 x 4	10.	55.56 ÷ 6	11.	230.91 ÷ 3
12.	877.535 ÷ 5				

Answers on page 35

FRACTIONS

The numbers $\frac{1}{2}, \frac{3}{8}, \frac{7}{24}$ are all known as fractions. In some instances, such as this unit, we are required to deal with these without the use of a calculator.

Simplification of Fractions

By simplification of fractions we mean expressing a fraction in its “lowest terms”, i.e. writing the numerator and denominator with the smallest possible numbers.

Example (1)

Simplify $\frac{6}{9}$ $\frac{\text{numerator}}{\text{denominator}}$

The fraction may be written as $\frac{2 \times \cancel{3}^{1}}{3 \times \cancel{3}_{1}} = \frac{2}{3}$

Numerator and denominator can now be divided by 3, known as cancelling.

i.e. $\frac{6}{9}$ can be simplified to $\frac{2}{3}$

As both fractions have the same value, they are called **equivalence fractions**.

ADDITION AND SUBTRACTION

When adding or subtracting fractions, they must have the same denominators. To achieve this we must form equivalence fractions. For any given fraction, there are lots of equivalence fractions which can be formed.

Example (2)

$$\frac{3}{4} = \frac{?}{8}$$

The original denominator has been multiplied by 2, so we must do the same to the numerator.

$$\frac{3}{4} \overset{\times 2}{\underset{\times 2}{=}} \frac{6}{8}$$ (would also equal $\frac{15}{20}$ by multiplying by ?)

RULES FOR ADDING OR SUBTRACTING FRACTIONS

- Make sure all fractions have the same denominator (use equivalence fractions if necessary).
- Express as a single fraction.
- Add or subtract terms in the numerator.

Example (3)

$$\frac{2}{7} + \frac{4}{7}$$

Denominators are the same so express as a single fraction

$$= \frac{2+4}{7}$$

Add terms in the numerator

$$= \frac{6}{7}$$ (we have added 2 sevenths and 4 sevenths)

If the denominators are different, we have to form equivalence fractions using the LOWEST COMMON DENOMINATOR (L C D). This is the smallest number which each denominator will divide into.

Example (4)

$$\frac{4}{5} - \frac{3}{4}$$

The LCD is 20, so forming equivalence fractions :

$$\frac{4}{5} - \frac{3}{4} = \frac{16}{20} - \frac{15}{20} = \frac{16-15}{20} = \frac{1}{20}$$

SAQ6

Write each of the following as a fraction in its simplest form.

1. $\frac{2}{9} + \frac{4}{9}$
2. $\frac{3}{4} - \frac{1}{2}$
3. $\frac{9}{8} + \frac{1}{5}$
4. $\frac{7}{8} - \frac{1}{4}$
5. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$
6. $\frac{4}{7} + \frac{11}{21}$

Answers on page 36

MULTIPLICATION

When multiplying fractions :

- Multiply the numerators together.
- Multiply the denominators together.
- Cancel if possible.

Example (5)

$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ (no cancelling possible)

Example (6)

$\frac{4}{3} \times \frac{5}{7} = \frac{20}{21}$ (no cancelling possible)

Example (7)

$\frac{3}{4} \times \frac{2}{5} = \frac{\overset{3}{\cancel{6}}}{\underset{10}{\cancel{20}}} = \frac{3}{10}$ (cancel by 2)

SAQ7

Write each of these as as a single fraction in its simplest form.

1. $\frac{3}{7} \times \frac{4}{5}$ 2. $\frac{1}{2} \times \frac{4}{9}$ 3. $\frac{5}{7} \times \frac{6}{11}$

4. $\frac{5}{4} \times \frac{3}{8}$ 5. $\frac{3}{4} \times \frac{2}{5} \times \frac{5}{6}$

Answers on page 36

DIVISION

When dividing :

- Invert the fraction you are dividing by (i.e. turn it upside down)
- Change the sign from ÷ to x
- Multiply the fractions together.

Example (8)

$$\frac{5}{8} \div \frac{3}{4}$$

$$= \frac{5}{8} \times \frac{4}{3}$$

$$= \frac{\overset{5}{\cancel{20}}}{\underset{6}{\cancel{24}}} = \frac{5}{6}$$ (cancelling by 4)

Example (9)

$$2 \div \frac{1}{2}$$

$= \frac{2}{1} \times \frac{2}{1}$ (note that the whole number 2 is written as the fraction $\frac{2}{1}$

$= \frac{4}{1} = 4$ (we know this is true because there are 4 halves in 2)

Example (10)

$\frac{5}{4} \div \frac{2}{3} = \frac{5}{4} \times \frac{3}{2} = \frac{15}{8}$ called an improper fraction which may be written

$$\frac{15}{8} = 1\frac{7}{8}$$

SAQ8

Write each of these as a single fraction in its simplest form.

1. $\frac{7}{16} \div \frac{4}{5}$
2. $\frac{8}{11} \div \frac{2}{3}$
3. $7 \div \frac{1}{7}$

Answers on page 36

SQUARES, SQUARE ROOTS AND RECIPROCALS

These will be required more in the following units in connection with electrical formulae such as

$$P = \frac{V^2}{R} \qquad P = I^2R \quad \text{and} \quad \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

when a calculator will be used but it is useful to introduce them now with simple numbers.

SQUARES

If we multiply a number by itself, we say that it is squared.

That is, the square of 3 written as $3^2 = 3 \times 3 = 9$.

5^2 called five squared would be $5 \times 5 = 25$

5^3 called five cubed would be $5 \times 5 \times 5 = 25 \times 5 = 125$

SQUARE ROOTS

The opposite operation to squaring is to find the square root of a number.

Since $3 \times 3 = 3^2 = 9$ and $-3 \times -3 = 9$

Then the square root of 9 written as $\sqrt{9} = 3$
In practical applications such as in the formulae mentioned earlier, the positive sign is taken and we accept 3 as $\sqrt{9}$

Clearly $\sqrt{25} = 5$

RECIPROCALS

The reciprocal of the number 2 is $\frac{1}{2}$

The reciprocal of the number 3 is $\frac{1}{3}$

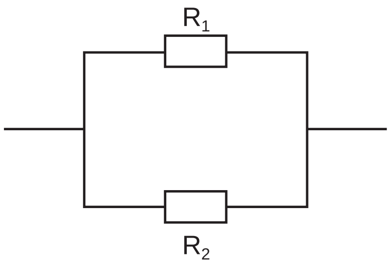
The reciprocal of $\frac{7}{9}$ is $\frac{9}{7}$

Reciprocals and addition of fractions are used when considering electrical resistances in parallel:

Example

If resistors $R_1 = 4\Omega$ and $R_2 = 8\Omega$ are connected in parallel

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$



$$\frac{1}{R_T} = \frac{1}{4} + \frac{1}{8} = \frac{2}{8} + \frac{1}{8} = \frac{2+1}{8} = \frac{3}{8}$$

=> $\frac{1}{R_T} = \frac{3}{8}$

=> R_T equals the reciprocal of $\frac{3}{8} = \frac{8}{3}\,\Omega$

SAQ9

Try these where the answers to the square roots are all whole numbers :

1. 7^2
2. 25^2
3. 6^2
4. 6^3
5. 12^2
6. $\sqrt{144}$
7. $\sqrt{169}$
8. $\sqrt{225}$
9. Write down the reciprocals of 2, 5, 8, $\frac{1}{4}$ and $\frac{1}{10}$

10 Resistors of 5 Ω and 10 Ω are connected in parallel. Calculate the total resistance leaving your answer as a fraction.

Answers on page 36

PERCENTAGES

Percentages are just fractions with a denominator of 100 and are a useful method of comparing quantities when expressed in this way. For example, two students take an examination. One examination is marked out of 65 and the first student scores 50 marks.
The second examination is marked out of 75 and the second student scores 60.
Which student has obtained the better result?

Not obvious from these figures but converting to percentages we find the first student has scored almost 77 % and the second 80 %.

CHANGING FRACTIONS TO PERCENTAGES

To change a fraction to a percentage, multiply it by 100 and write the percentage symbol which you know is %.

EXAMPLES

$\frac{3}{5}$ as a percentage will be $\frac{3}{5} \times \frac{100\%}{1} = \frac{300}{5} = 60\%$

$\frac{7}{40}$ as a percentage will be $\frac{7}{40} \times \frac{100}{1} = \frac{35}{2} = 17.5\%$

(recall multiplying fractions and cancelling common factors in numerator and denominator , in this case 20)

$1\frac{1}{8}$ this is a mixed number being one and an eighth and has to be changed to an improper fraction before multiplication

i.e. $1\frac{1}{8} = \frac{8}{8} + \frac{1}{8} = \frac{9}{8}$

Hence $1\frac{1}{8}$ as a percentage will be $\frac{9}{8} \times \frac{100}{1} = \frac{9}{2} \times \frac{25}{1} = \frac{225}{2} = 112.5\%$

CHANGING DECIMAL FRACTIONS TO PERCENTAGES

The same rule applies - multiply the decimal fraction by 100 i.e. move the decimal point two places to the RIGHT and write the symbol for percentage %

Examples

0.135 as a percentage will be 0.135 x 100 = 13.5 %

0.08 as a percentage will be 0.08 x 100 = 8 %

1.125 as a percentage will be 1.125 x 100 = 112.5 %

2 as a percentage will be 2.0 x 100 = 200 %

(the implied decimal point not usually shown in a whole number.)

The fraction of $1\frac{1}{8}$ met earlier could have been written as 1.125 and then converted to a percentage

SAQ10

Change these into percentages :

(1) $\frac{1}{2}$	(2) $\frac{1}{4}$	(3) $\frac{3}{4}$	(4) $\frac{4}{5}$	(5) $\frac{3}{10}$
(6) $\frac{7}{20}$	(7) $\frac{5}{8}$	(8) $\frac{5}{12}$	(9) $1\frac{1}{2}$	(10) $3\frac{3}{10}$
(11) 0.66	(12) 0.455	(13) 0.08	(14) 2.39	(15) 0.025

Answers on page 37

CHANGING PERCENTAGES TO FRACTIONS

To convert a percentage to a fraction divide by 100

Examples

Converting to common fractions :

$$60\ \% = \frac{60}{100} = \frac{3}{5} \quad \text{(cancel by common factor 20)}$$

$$12\frac{1}{2}\ \% = \frac{12\frac{1}{2}}{100} \times \frac{2}{2} = \frac{25}{200} = \frac{1}{8} \quad \text{(cancel by common factor 25)}$$

Note that we converted $12\frac{1}{2}$ to halves by multipling the numerator by 2 so also multplied the denominator by 2 to obtain the equivalence fraction)

Converting to decimal fractions :

$$45\% = \frac{45}{100} = 0.45$$

Note that to divide by 100, move the ‘implied decimal point‘ two places to the LEFT.

$$175\% = \frac{175}{100} = 1.75$$

SAQ11

Change the following percentages to (a) a common fraction
(b) a decimal fraction

(1) 28 % (2) 150 % (3) 1.25 % (4) $37\frac{1}{2}$ %

(5) $33\frac{1}{3}$ % (6) 1 %

Answers on page 37

FINDING A PERCENTAGE OF A QUANTITY

EXAMPLE

15 % of £60

Method 1

$$15\,\%\ \text{of}\ £60 = \frac{\overset{3}{\cancel{15}}}{\underset{\underset{1}{\cancel{20}}}{\cancel{100}}} \times \frac{\overset{3}{\cancel{60}}}{1} = \frac{9}{1} = £9$$

(useful to simplify fractions first by cancelling if not using a calculator)

Method 2

15 % of £60 = 0.15 x 60 = £9

$$\text{Also}\ 12\frac{1}{2}\,\%\ \text{of}\ £64 = \frac{12\frac{1}{2}}{100} \times \frac{2}{2}\ \text{of}\ £64 = \frac{25}{200} \times 64 = \frac{1}{8} \times 64 = £8$$

$$\text{Or}\quad 12\frac{1}{2}\,\%\ \text{of}\ £64 = \frac{12.5}{100} \times 64 = 0.125 \times 64 = £8$$

(less easy if no calculator !)

SAQ12

Calculate :

(1) 25 % of £8.24

(2) 60 % of £72.00

(3) $66\frac{2}{3}$ % of £22.50

(4) $2\frac{1}{2}$ % of £364

Find ? % in questions (5) and (6) :

(5) ? % of 80 = 20

(6) ? % of 48 = 6

Answers on page 37

EXPRESSING ONE QUANTITY AS A PERCENTAGE OF ANOTHER

Make a fraction of the two quantities, multiply by 100 and write % symbol

Examples

(1) An apprentice scored 27 out of 45 in a numeracy test. What was his mark as a percentage ?

Fraction $= \dfrac{27}{45}$

Percentage mark $= \dfrac{\cancel{27}^{\,3}}{\cancel{45}_{\,\cancel{5}\,1}} \times \dfrac{\cancel{100}^{\,20}}{1} = 60\%$

(2) What percentage is 20p of £5 ?

In this case, the two quantities must be expressed in the same units :

Fraction $= \dfrac{20}{500}$

Percentage $= \dfrac{20}{500} \times \dfrac{100}{1} = 4\%$

PERCENTAGE CHANGE (increase or decrease as a %)

$$\text{Percentage increase (or decrease)} = \frac{\text{Actual increase (or decrease)}}{\text{Original figure}} \times \frac{100}{1}$$

(3) A car is reduced in price from £12500 to £9000. Find the percentage price reduction.

Actual decrease = £12500 - £9000 = £3500

Percentage decrease $= \dfrac{3500}{12500} \times \dfrac{100}{1} = \dfrac{7}{1} \times \dfrac{4}{1} = 28\%$

SAQ13

What percentage is :

(1) 18 of 25 (2) £1.45 of £5 (3) 58p of £116

(4) A speed of 40 mph is increased to 50 mph. Calculate the % increase in speed.

(5) A voltmeter reads a supply voltage of 235.2 volts. If the supply voltage was actually 240 volts, find the percentage error in the reading.

Answers on page 37

ALGEBRA / EVALUATION OF FORMULAE

When using formulae, symbols or letters are used in place of numbers. Even with letters we still follow the pattern of arithmetic.

$a + b$	means	a plus b
$a - b$	means	a minus b
$a \times b$	means	a multiplied by b usually written as ab
$\frac{a}{b}$	means	a divided by b written $a \div b$
Note that 5c	means	5 multiplied by c

EVALUATION OF FORMULAE

A formula is simply a set of instructions in algebra which tells you how to carry out a calculation.

Examples

(1) The formula for finding the volume V of a rectangular box of length l, breadth b, and height h is :

$V = lbh$

Determine the volume of the box if $l = 8$cms, $b = 10$cms and $h = 9$cms

$V = lbh = 8 \times 10 \times 9 = 80 \times 9 = 720 \text{ cm}^3$ (cm^3 = cubic centimetres)

(2) Ohms Law states $I = \frac{V}{R}$

If $R = 5\Omega$ and $V = 75$ volts then $I = ?$ amps

$I = \frac{V}{R} = \frac{75}{5} = 15$ amps

(3) The circumference C of a circle is given by the formula

$C = 2\pi r$ where π has the value $\frac{22}{7}$ and r is the radius

Calculate the circumference of a circle of radius $r = 14$ cms

$C = 2\pi r = 2 \times \frac{22}{7} \times 14 = 2 \times 22 \times 2 = 88$ cms

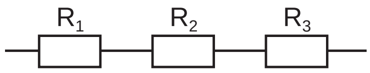
(4) Power $P = I^2 R$ watts

If $I = 15$ amps and $R = 4\Omega$, find the power dissipated in watts.

$P = I^2 R = 15^2 \times 4 = 225 \times 4 = 900$ watts

(5) The total resistance R in a series circuit is given by

$R_T = R_1 + R_2 + R_3$



where $R_1 = 0.75\Omega$, $R_2 = 0.05\Omega$ and $R_3 = 0.25\Omega$

$R_T = R_1 + R_2 + R_3 = 0.75 + 0.05 + 0.25$ (recall 0.75
0.05
<u>0.25</u>
1.05)

=> $R_T = 1.05\Omega$

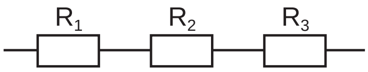
SAQ14

(1) Using Ohms Law $I = \frac{V}{R}$ determine the current I when :

(a) $V = 500$ volts and $R = 1000\Omega$
(b) $V = 240$ volts and $R = 12\Omega$
(c) $V = 120.4$ volts and $R = 7\Omega$

(2) The total resistance in a series circuit R is given by :

$R_T = R_1 + R_2 + R_3$



If $R_1 = 7.5\Omega$, $R_2 = 15\Omega$ and $R_3 = 0.075\Omega$ then $R_T = ?$

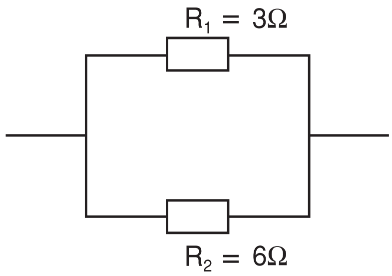
(3) Use $P = I^2 R$ to determine the power P dissipated when :

(a) $I = 9$ amps and $R = 4\Omega$
(b) $I = 14$ amps and $R = 5\Omega$

(4) If resistors R_1 and R_2 are in parallel :

$\frac{1}{R_T} = \frac{I}{R_1} + \frac{1}{R_2}$ *then* $\frac{1}{R_T} = ?$

$R_T = ?$



(5) The formula for converting degrees Celcius to degrees Fahrenheit is :

$$F = \frac{9C}{5} + 32$$

Convert the following degrees C to degrees F

(a) 10 (b) 100 (c) 20 (d) 0

(6) The impedance Z in a circuit is given by the formula :

$Z = \sqrt{R^2 + X_c^2}$ where X_c is the reactance and R is the resistance

Find Z when
(a) $R = 3$ and $X_c = 4\Omega$
(b) $R = 5$ and $X_c = 12\Omega$
(c) $R = 8$ and $X_c = 15\Omega$
(d) $R = 24$ and $X_c = 7\Omega$

Answers on page 38

TRANSPOSITION OF FORMULAE

Consider the Ohm's Law formula $I = \frac{V}{R}$

As written here, I is called the subject of the formula, and we would use it as you have earlier to calculate I given the values for V and R..
However, if we had to calculate V given values for R and I, then the formula would have to be rearranged before we substituted any values. That is we would have to make V the subject.

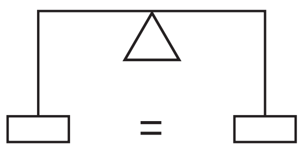
This process is known as changing the subject or transposing the formula.

There are so many different types of formula that it is difficult to state a set of rules which apply in every case but the following guidelines should be used where possible.

Guidelines for transposing Formulae

- Whatever is done to one side of the formula must be done to the other
- Decide how the various symbols are connected, then do the opposite to separate them i.e. if two symbols are multiplied, we divide; if a symbol is squared, take the square root.
- In general, it is wise to remove fractions and brackets.
- Following these guidelines, isolate the required subject on one side of the formula

It may be helpful to think of a weighing balance
It will stay perfectly balanced as long as you add
or take away the same weight from both sides.



A formula will be true provided the same operation is carried out on both sides as the following examples illustrate.

EXAMPLES

(1) Make V the subject of the formula $I = \frac{V}{R}$

V is divided by R so multiply both sides by R :

$$I = \frac{V}{R}$$

$$IR = \frac{V\cancel{R}}{\cancel{R}}$$

=> $IR = V$ cancelling the R s

or $V = IR$ as it is usual to put the subject on the left hand side and the formula is now suitable for finding V given values for I and R.

(2) Given $S = \frac{d}{t}$ make t the subject

Remove fractions - multiply both sides by t

$$St = \frac{d\cancel{t}}{\cancel{t}}$$

=> $St = d$

or $d = St$

=> $\frac{d}{S} = t$ or $t = \frac{d}{S}$

(3) Make x the subject of the formula

$y = mx + c$

First isolate mx by subtracting c from both sides (because mx is added to c)

$y - c = mx + c - c$

$y - c = mx$

Now divide both sides by m to isolate x (because x is multiplied by m)

$$\frac{y - c}{m} = \frac{\cancel{m}x}{\cancel{m}}$$

$\frac{y - c}{m} = x$ => $x = \frac{y - c}{m}$

(4) Make I the subject of the formula $P = I^2 R$

Divide both sides by R to isolate I^2

$$\frac{P}{R} = \frac{I^2 R}{R}$$

$$\Rightarrow \quad \frac{P}{R} = I^2$$

Take the square root of both sides :

$$\sqrt{\frac{P}{R}} = I$$

Or $\quad I = \sqrt{\frac{P}{R}}$

Transposition and evaluation are often both required :

(5) Given $v = u - ft$ where $v = 8$, $u = 28$ and $f = 4$ then $t = ?$

You may prefer to substitute the values first :

$8 = 28 - 4t$

$8 - 28 = -4t$

$-20 = -4t$

$$\frac{-20}{-4} = \frac{-4t}{-4}$$

$\Rightarrow \quad 5 = t$ (recall rule of signs in division - minus divide by minus is a plus)

so $t = 5$

If transposition is done first :

$v - u = -ft$ (subtracting u from both sides)

$\frac{v - u}{-f} = \frac{-ft}{-f}$ (dividing both sides by -f)

$$\frac{v - u}{-f} = t$$

So $t = \dfrac{v - u}{-f}$

Now substitute into this formula

=> $t = \dfrac{8 - 28}{-4} = \dfrac{-20}{-4} = 5$ (rule of signs again !)

SAQ15

Rearrange the following so the symbol in brackets is made the subject of the formula :

1. $V = IR$ (R)
2. $V = u + at$ (a)
3. $P = I^2 R$ (R)
4. $K = M - 10N$ (N)
5. $P = \dfrac{V^2}{R}$ (R)
6. $A = \pi r^2$ (r)
7. $X = 2\pi f$ (f)
8. $S = ut + gt^2$ (g)
9. If the power in a circuit is 64 watts and the resistance is 4Ω, determine the current I using the formula

   $P = I^2 R$

10 $a = 9b + 3c$ where $a = 93$ and $b = 7$ then $c = ?$

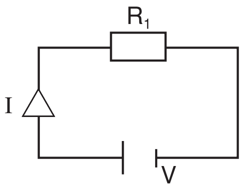
Answers on page 38

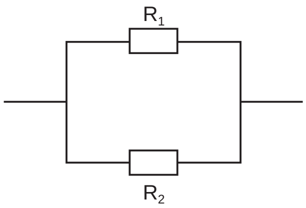
In the next unit you will meet transposition and evaluation of more complex formulae with more realistic numbers.
The same ideas apply in the transposition and also a calculator will be used.
To monitor how you have progressed in this unit, there is a final multiple choice test for you to try when you have completed all the SAQs.

MULTIPLE CHOICE TEST

This multiple choice test is designed to give you a quick way of checking your progess in the introductory unit. Try it when you have completed and checked all the SAQs.
As each question has only one correct answer, you may be able to complete some of the questions by eliminating possible answers without working the question out in full.
Circle the white number to select your answer and of course you should use a separate sheet of paper for your working.

		1	2	3	4
Q1	**432 + 986 + 218 =**	16	1636	1618	1640
Q2	**3927 - 2146 =**	1781	1771	1792	1780
Q3	**499 x 8 =**	3982	3978	3992	3998
Q4	**6468 ÷ 7 =**	924	904	932	928
Q5	**£325.78 + £725.56 + £19.10 =**	£998.44	£1060.64	£1080.64	£1070.44
Q6	**£98.50 - £37.20 =**	£61.30	£70.40	£61.40	£50.30
Q7	**£ 39.67 x 6 =**	£238.02	£240.02	£236.12	£238.42
Q8	**£456.35 ÷ 5 =**	£90.72	£89.27	£93.72	£91.27
Q9	**14^2 =**	186	196	166	176
Q10	**5^3 =**	15	225	125	105
Q11	$\frac{1}{2} + \frac{1}{3} =$	$\frac{6}{5}$	$\frac{2}{5}$	$\frac{5}{6}$	$\frac{1}{5}$
Q12	$\frac{3}{2} - \frac{5}{4} =$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{5}{12}$	$\frac{2}{4}$
Q13	$\sqrt{0.64}$	0.16	0.46	0.8	0.08
Q14	$12\frac{1}{2}$% of £80 =	£8	£12	£9	£10
Q15	300 = ? % of 1200	35%	32%	25%	28%

		1	2	3	4
Q16	$\frac{4}{7} \times \frac{3}{5} =$	$\frac{12}{35}$	$\frac{7}{12}$	$1\frac{1}{2}$	$\frac{20}{21}$
Q17	$\frac{3}{4} \div \frac{4}{3}$	1	$\frac{9}{16}$	$\frac{1}{2}$	$\frac{7}{12}$
Q18	$\frac{?}{8} = \frac{12}{32}$? =	3	5	4	6
Q19	? % of 125 = 25	25%	21%	22 %	20%
Q20	$\sqrt{3 \times 48}$	9	12	13	10.5
Q21	Convert 64 % to a fraction	$\frac{7}{9}$	$\frac{5}{6}$	$\frac{4}{7}$	$\frac{16}{25}$
Q22	Convert $\frac{5}{8}$ to a percentage	52.5%	62.5%	62%	60%
Q23	Convert $4\frac{1}{4}$ to a decimal	4.35	4.15	4.25	4..45
Q24	v = u + ft t = 5, u = 20 and f = 4 v = ?	100	45	40	85
Q25	m = 30n - 2p n = 5, m = 132 p = ?	12	28	9	18
Q26	If the total resistance R_T in a series circuit is given by : $R_T = R_1 + R_2 + R_3$ and $R_1 = 0.45\Omega$, $R_2 = 0.05\Omega$, $R_3 = 0.8\Omega$ then R_T = ?	1.25Ω	0.95Ω	1.2Ω	1.3Ω
Q27	Ohms Law states $I = \frac{V}{R}$  If V = 240 watts and R = 40Ω then I = ? amps	6.5	6	8	4

		1	2	3	4
Q28	Ohms Law states $I = \frac{V}{R}$ If $R = 8\Omega$ and $I = 12$ amps then $V =$? volts	88	86	96	106
Q29	Ohms Law again ! If $I = 5$ amps and $V = 250$ volts then $R = ?\Omega$	25Ω	50Ω	40Ω	60Ω
Q30	 If the total resistance R_T in a parallel circuit is given by $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$ and $R_1 = 4\Omega$, $R_2 = 3\Omega$ then $\frac{1}{R_T} =$?	$\frac{1}{2}$	$\frac{2}{7}$	$\frac{3}{4}$	$\frac{7}{12}$
Q31	In question 30 $R_T = ?\Omega$	1	$\frac{7}{2}$	$\frac{12}{7}$	$\frac{4}{3}$
Q32	In Ohms Law $I = \frac{V}{R}$ if V is tripled and the resistance R is tripled, what will happen to the current I ?	up $\frac{1}{3}$	same	tripled	halved
Q33	$P = I^2 R$ watts If $I = 12$ amps and $R = 5\Omega$ then power dissipated is $P =$? watts	720	300	620	120
Q34	$P = I^2 R$ watts If $P = 400$ watts and $I = 10$ amps Then $R = ?\Omega$	20	40	4	2
Q35	Again, $P = I^2 R$ watts If $P = 576$ watts and $R = 4\Omega$ then $I =$? amps	13	12	72	24

Answers to SAQ's
and
Multiple Choice Test

ANSWERS

SAQ1

1.	7	2.	27	3.	1	4.	-9
5.	5	6.	27	7.	399	8.	933
9.	214	10.	56	11.	4655	12.	444

SAQ2

1.	45	2.	24	3.	54	4.	16
5.	42	6.	25	7.	24	8.	-48
9.	-56	10.	6	11.	64	12.	81

SAQ3

1.	729	2.	432	3.	1368
4.	1974	5.	43960	6.	10829

SAQ4

1.	27	2.	147	3.	972	4.	-2
5.	375	6.	-2	7.	41	8.	412
9.	703	10.	434	11.	651	12.	6849

SAQ5

1.	17.19	2.	1550.88	3.	3.53	4.	2.482
5.	1686.64	6.	272.25	7.	36.4	8.	397.2
9.	873.48	10.	9.26	11.	76.97	12.	175.507

SAQ6

1. $\frac{6}{9} = \frac{2}{3}$
2. $\frac{1}{4}$
3. $\frac{53}{40} = 1\frac{13}{40}$
4. $\frac{5}{8}$
5. $\frac{13}{12} = 1\frac{1}{12}$
6. $\frac{23}{21} = 1\frac{2}{21}$

SAQ7

1. $\frac{12}{35}$
2. $\frac{4}{18} = \frac{2}{9}$
3. $\frac{30}{77}$
4. $\frac{15}{32}$
5. $\frac{1}{4}$

SAQ8

1. $\frac{35}{64}$
2. $\frac{24}{22} = \frac{12}{11} = 1\frac{1}{11}$
3. 49

SAQ9

1. 49
2. 225
3. 36
4. 216
5. 144
6. 12
7. 13
8. 15
9. $\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{8}$, 4 and 10
10. $\frac{1}{R_T} = \frac{1}{5} + \frac{1}{10} = \frac{2}{10} + \frac{1}{10} = \frac{3}{10}$

    => $R_T = \frac{10}{3}\ \Omega$

SAQ10

1	50 %	2	25 %	3	75 %	4	80 %
5	30 %	6	35 %	7	$62\frac{1}{2}$% or 62.5 %	8	$41\frac{2}{3}$ % or 41.667%
9	150 %	10	330 %	11	66 %	12	45.5 %
13	8 %	14	239 %	15	2.5 %		

SAQ11

1	(a) $\frac{7}{25}$	(b) 0.28	2	(a) $\frac{3}{2}$	(b) 1.5
3	(a) $\frac{1}{80}$	(b) 0.0125	4	(a) $\frac{3}{8}$	(b) 0.375
5	(a) $\frac{1}{3}$	(b) 0.333....	6	(a) $\frac{1}{100}$	(b) 0.01

SAQ12

(1)	£2.06	(2)	£43.20	(3)	£15	(4)	£9.10
(5)	25 %	(6)	12.5 %				

SAQ13

(1)	72 %	(2)	29 %	(3)	0.5 %
(4)	28 %	(5)	2 %		

SAQ14

1 (a) $\frac{1}{2}$ amp (b) 20 amps (c) 17.2 amps 2. 22.575Ω

3 (a) 324 watts (b) 980 watts

4. $$\frac{1}{R_T} = \frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

$$\Rightarrow R_T = 2\,\Omega$$

5 (a) 50 (b) 212 (c) 68 (d) 32

6 (a) 5 (b) 13 (c) 17 (d) 25

SAQ15

1. $R = \frac{V}{I}$ 2. $a = \frac{v - u}{a}$

3. $R = \frac{P}{I^2}$ 4. $N = \frac{M - K}{10}$

5. $R = \frac{V^2}{P}$ 6. $r = \sqrt{\frac{A}{\pi}}$

7. $f = \frac{X}{2\pi}$ 8. $g = \frac{s - ut}{t^2}$

9. $I^2 = \frac{P}{R}$

$$\Rightarrow I = \sqrt{\frac{P}{R}} = \sqrt{\frac{64}{4}} = \sqrt{16} = 4 \text{ amps}$$

10. $a - 9b = 3c$

$$\Rightarrow \frac{a - 9b}{3} = c \Rightarrow c = \frac{a - 9b}{3} = \frac{93 - 63}{3} = \frac{30}{3} = 10$$

MULTIPLE CHOICE TEST

Q1	2	Q2	1	Q3	3	Q4	1	Q5	4
Q6	1	Q7	1	Q8	4	Q9	2	Q10	3
Q11	3	Q12	2	Q13	3	Q14	4	Q15	3
Q16	1	Q17	2	Q18	1	Q19	4	Q20	2
Q21	4	Q22	2	Q23	3	Q24	3	Q25	3
Q26	4	Q27	2	Q28	3	Q29	2	Q30	4
Q31	3	Q32	2	Q33	1	Q34	3	Q35	2